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Corrigendum

Corrigendum to "Integrals of motion in the many-body localized phase" [Nucl. Phys. B 891 (2015) 420–465]

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Abstract

We correct a small error in our article *Integrals of motion in the many body localized phase* [1]. The correction does not alter the main result regarding the convergence of the perturbative expansion for integrals of motion in forward approximation, but reduces the estimate of the radius of convergence by a numerical factor of roughly $\simeq 1.79$.

In Ref. [1] we associated to any diagram with N interaction vertices a minimal number of 'effective paths' that result from the integration procedure described in Section 5. We stated that this minimal number grows *sub-exponentially* with N, which relied on computing this number for diagrams d with a maximally branched geometry. However, the latter was erroneously determined to be sub-exponential in N in Eq. (C.6). The corrected version of Eq. (C.6) (counting the number of effective paths associated to either of the two rooted trees of the diagram) instead reads:

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$$\prod_{l=1}^{L(N)-1} (l+1)^{2\cdot 3^{L(N)-1-l}} = \exp\left(3^{L(N)} \sum_{k=1}^{\infty} \frac{2\log k}{3^k} - \frac{2}{3}\sum_{k=0}^{\infty} \frac{\log(k+L(N)+1)}{3^k}\right), \tag{1}$$

with $L(N) = \log(N+1)/\log 3$. Using that $2\sum_{k=1}^{\infty} 3^{-k}\log(k) = 0.29$, one finds that the minimal number of effective paths $|\mathcal{P}|$ for diagrams with this geometry, which is the square of the above, scales as:

$$|\mathcal{P}| = \exp\big[0.58\,N + o(N)\big], \tag{2}$$

which should replace the estimate in Eq. (C.7).

Among all the possible geometries of diagrams with a fixed number of interactions N, the maximally branched geometry is the one that maximizes the number of effective paths. Thus, Eq. (2) is an upper bound for the average $\overline{|\mathcal{P}(d)|}$ introduced in Eq. (55), which we also expect to have an exponential scaling in N:

$$\overline{\mathcal{P}(d)} \sim \exp\big[\alpha\,N + o(N)\big] \tag{3}$$

with $0 < \alpha < 0.58$. Accounting for this correction, the total number of effective paths of length N, $\mathcal{N}_N$ in Eqs. (59), is modified accordingly:

$$\mathcal{N}_N \to e^{\alpha N}\mathcal{N}_N. \tag{4}$$

Since the additional factor is only exponential in N, the conclusion about the convergence of the construction of integrals of motion for small enough interactions is unaltered. The effect of the correction is to slightly diminish the radius of convergence of the construction.

The precise effect of this correction depends on the relative weight of the effective paths and their mutual interference.

If we make the simplifying assumption that the effective paths associated to the same diagram can be treated as independent random variables, the sum $S(d)$ in Eq. (53) is dominated by the largest term, and the factor $\overline{|\mathcal{P}(d)|}$ enhances the tail of the distribution of $\mathcal{A}^{(\alpha)}_{\mathcal{I},\mathcal{J}}$ as compared to the tail corresponding to a single path weight, see Eq. (58). The above discussed correction would thus modify by a factor e^{α} the numerical constant C in Eq. (93). (Note, however, that this constant C was already subject to uncertainty, see Eq. (85), due to the approximations going into the estimate of $\mathcal{N}_N$.) Approximating $e^{\alpha} \approx e^{0.58} = 1.79$ we find that the result of Eq. (93) holds with the following uncertainty on C:

$$18.97 < C < 36.25. \tag{5}$$

The above assumption neglects, however, that the effective paths associated to a given diagram are not independent. Indeed, they involve the same energy variables in the denominators, but in different combinations. These correlations might be relevant when computing the large deviations for $S(d)$. In fact there could be disorder realizations in which all the energy variables are simultaneously small, in such a way that there is no dominant effective path. In an extreme case, all $\tilde{\omega}_\Gamma$ contributing to $S(d)$ might happen to be of the same order of magnitude and atypically large. These contributions will come with different signs and partially cancel, which counteracts the enhancement of the total amplitude. To estimate an upper bound for the effect of the exponential number of effective paths on the constant C we neglect those partial cancellations, and assume that the diagrams dominating the tails of $S(d)$ are such that essentially all effective paths add up constructively with comparable weights. Under this extreme scenario, the large deviations of $S(d)$ would be given in terms of those of a single path weight $\tilde{\omega}$ by setting $S(d) \sim \overline{|\mathcal{P}(d)|}\tilde{\omega}$. In this approximation, Eq. (93) is recovered with the substitution $\lambda \to \lambda e^{\alpha}$. This shifts the estimated interval for C in Eq. (5) only by a logarithmic factor.

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References

[1] V. Ros, M. Müller, A. Scardicchio, Nucl. Phys. B 891 (2015) 420.